Power Factor Calculation by The Finite Element Method.

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Analytical methodology remains the most important daily work tool of motor designers. The use of analytical tools is important because it reduces calculation time-especially during optimization. However, there are some important phenomena that are not always evaluated, particularly when ECM (Equivalent Circuit Models) are involved. For these,numerical methods derive the best results. The use of finite element analysis is very important because it provides detailed simulation and is more accurate regarding saturation and field distribution. There are numerous examples that show the importance of using finite element analysis to evaluate the performance of electrical motors.

Commonly, the power factor is used worldwide to quantify and to tax the real and reactive power of electrical systems. Its definition needs to be redefined to systems having nonsinusoidal currents or voltage waveform. The deviances of ideal conditions can lead to mistakes in measurement and taxing.

Traditionally, systems that consume alternative power, consume both the real power (P) and reactive power (Q), fed by the network, if pure sine waves are involved (voltage and currents). The vector sum of real and reactive power is the apparent power (S). The power factor PF of an electric motor is defined as the ratio of its real power in Watts to the apparent power in VA. The presence of reactive power causes the real power (or useful power) to be less than apparent power, consequently induction motors have power factor less than 1.

Motors with low power factors are an undesirable load on the network. The power factor of a single-phase induction motor can be easily obtained by tests and confirmed by simulations using the finite element method. The level of saturation can distort the current curve and lead to false values of power factor if the prior definition is taken.
In this example the finite element method is used to evaluate the performance of a small industrial capacitor connected to a singlephase induction motor. A method to extract the input power of a motor calculated from post-treatment of a finite element model, including consideration of saturation and harmonics, is proposed hereafter. A new factor, called a displacement factor, is then deducted and calculated. Results of simulations are compared to experimental ones. This work intends to be educational and gives enough information to engineers who are not used to performing a power factor calculation using the finite element method.

Power Factor Definition

When voltages and currents are pure single sine waveforms, power factor PF is defined as the rate of the real power P and the apparent power S consumed by machines or devices. The signals varying in time may be periodical and of the same frequency. The product of the signals gives the instantaneous value of the power. The average value of this product is the real power P. Taking V_i and I_i as the instantaneous value of input voltage and current varying in time, as well as V_{RMS} and I_{RMS} their rms values, the power factor is obtained with (1).

$$PF = \frac{P}{S} = \frac{\frac{1}{T}\int_0^T V_i(t)\, I_i(t)\, dt}{V_{RMS} * I_{RMS}} \qquad (1)$$

If the input voltage V is defined as $V(t)=V_P \sin(\omega t)$ and the input current as $I(t)=I_P \sin(\omega t-ø)$ the active power is obtained with (2). Voltage and current have only the fundamental component of frequency f and $\omega=2.\pi.f$. V_P and I_P are the peak values of voltage and current. The angle Ø is the phase angle difference between the current and voltage waveform as shown in Fig.1.

$$P = \frac{1}{T}\int_0^T V_P \sin(\omega t)\, I_P \sin(\omega t - \phi)dt \qquad (2)$$

If we take $a=\omega t$, $b=\omega t-\phi$, $a-b=\phi$, $a+b=2\omega t+\phi$ and additionally

$\sin(a).\sin(b)=0.5\cos(a-b) - 0.5\cos5(a+b)$ we have (3).

$$P = \frac{1}{T}\int_0^T V_P\, I_P\, (0.5 * \cos(\phi) - 0.5 * \cos(2\omega t + \phi))dt \qquad (3)$$

It is possible to show that:

$$\frac{1}{T}\int_0^T \cos(2\omega t + \phi)dt = 0 \qquad (4)$$

$$\frac{1}{T}\int_0^T \cos(\phi)dt = \cos\phi \qquad (5)$$

Then, with (4) and (5) in (3) we have (6).

$$P = \frac{1}{2} V_P\, I_P \cos\phi \qquad (6)$$

Consequently is possible to express the active power P with the equation (7).

$$P = \frac{V_P}{\sqrt{2}} \frac{I_P}{\sqrt{2}} \cos\phi \qquad (7)$$

As $I_{RMS}= I_P/ \sqrt{2}$ and $V_{RMS}= V_P/ \sqrt{2}$, replacing (7) in (1) we have $PF=\cos\phi$. This is the most academic and traditional model used to analyze the power factor and it is called *general form*. However this definition is true only for cases where the voltage and current are sinusoidal because the angle ϕ is always the same regardless if the passage by zero or the wave's peaks are used by the observer.

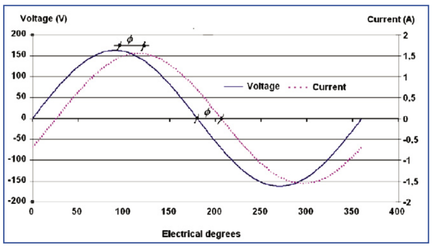

Fig. 1: Phase angle ϕ between current and voltage having the same frequency.

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Power Factor Calculation by The Finite Element Method. (continued)

Motor designers sometimes use the *direct model* to estimate the power factor. In this model the time of passage by zero is taken directly from curves obtained in simulations. This model should not be used when waveforms are charged of harmonics because a constant phase angle difference is not applicable.

We could see, for instance, actual time differences where the curves change direction and the waves peak occurs, which would lead to calculated values much different than experimental ones.
In a non-ideal case, voltage and current may have a nonsinusoidal waveform due to the use of inverters, converters, reactors, magnetic saturation and so forth. In this case the definition of power factor, as established above, is not correct.
Figure 2 shows the waveform of a sinusoidal input voltage and a saturated input current. It is possible to see that the angle of passage by zero $\phi 1$ and the one where we have the wave's peak $\phi 2$ is not the same. Here the definition of power factor, as established for sinusoidal inputs, is lost, thus creating the need to establish, a new definition.

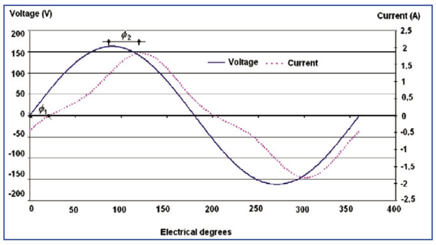

Fig.2. Different ϕ between current charged of harmonics and pure sine wave voltage.

First, it is necessary to show that the product of voltage versus harmonics current is null for all harmonics (only the product of components having the same frequency has an average value that is not null). For example, if the applied voltage is sinusoidal and the current is limited to a frequency of order 3, the equation (2) can be rewritten as (8).

$$P = \frac{1}{T}\int_0^T V_P \sin(\omega t)\, I_P \sin(3\omega t - \phi_3)dt \qquad (8)$$

If we now take a=ωt, b=3ωt- ϕ_3, a-b=-2ωt+ ϕ_3 and a+b=4ωt+ ϕ_3 we have (9).

$$P = \frac{1}{T}\int_0^T V_P\, I_P\, (0.5 * \cos(-2\omega t + \phi_3) - 0.5 * \cos(4\omega t + \phi_3))dt \qquad (9)$$

It is possible to show that:

$$\frac{1}{T}\int_0^T \cos(-2\omega t + \phi_3)dt = 0 \qquad (10)$$

$$\frac{1}{T}\int_0^T \cos(4\omega t + \phi_3)dt = 0 \qquad (11)$$

With (10) and (11) in (9) we have P=0, which means, in case if voltage and current do not have the same frequency, no active power is available.
If the input current has a fundamental and a third harmonic, $I(t)=I_P \sin(\omega t- \phi_1) + I_3 \sin(3\omega t- \phi_3)$, then the active power is defined as (12):

$$P = \frac{1}{T}\int_0^T V_P \sin(\omega t) * \{I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t - \phi_3)\}dt \qquad (12)$$

When the product of different frequencies will be null, the active power will be defined as (13). Only the fundamental of current carries the active power.

$$P = \frac{V_P}{\sqrt{2}}\frac{I_P}{\sqrt{2}}\cos\phi_1 \qquad (13)$$

The equation (13) says that to obtain the active power it is necessary to extract the fundamental of input current I_{P1} and its displacement ϕ_1 regarding the input voltage.
When voltage and current have only the fundamental frequency, cosϕ and cos ϕ_1 has the same meaning and are called *power factor*. However, when the current has several harmonics cos ϕ_1 has a different meaning and it is called *displacement factor*. General relations are summarized in Table I.

In the next section results of simulations and tests illustrate the theory presented.

	Pure sine waves (fundamental only)
Voltage	$u(t)=\hat{u}\times\cos(\omega t)=U\times\sqrt{2}\times\cos(\omega t)$
Current	$i(t)=\hat{i}\times\cos(\omega t+\phi)=I\times\sqrt{2}\times\cos(\omega t+\phi)$
Apparent power	$S=U\times I$
Apparent power	$S=\sqrt{P^2+Q^2}$
Active power	$P=\frac{1}{T}\int_{t_0}^{t_0+T}(U(t)\times I(t))dt$
Active power	$P=U\times I\times\cos(\phi)$
Note	ϕ is the phase difference between *u(t)* and *i(t)* and *cos*ϕ is called power factor
Reactive power	$Q=U\times I\times\sin(\phi)$
	Harmonics charged current
Voltage	$u(t)=\hat{u}\times\cos(\omega t)=U\times\sqrt{2}\times\cos(\omega t)$
Current	$i(t)=I_1\times\sqrt{2}\times\cos(\omega t+\phi_1)+I_2\times\sqrt{2}\times\cos(2\omega t+\phi_2)+....+I_n\times\sqrt{2}\times\cos(n\omega t+\phi_n)$
Apparent power	$S=\sqrt{U^2\times I^2}$; $U^2=U_1^2$; $I^2=I_1^2+I_2^2+I_3^2+...+I_n^2$
Apparent power	$S=\sqrt{P^2+Q^2+D^2}$
Active power	$P=\frac{1}{T}\int_{t_0}^{t_0+T}(U(t)\times I(t))dt$
Active power	$P=U\times I_1\times\cos(\phi_1)$
Note	ϕ_1 is the phase difference between *u(t)* and $i_1(t)$ and cosϕ_1 is called displacement factor
Reactive power	$Q=U\times I_1\times\sin(\phi_1)$
Deforming power	$D\Leftrightarrow D^2=U_1^2\times(I_1^2+I_2^2+I_3^2+...+I_n^2+...)=U_1^2\times I_h^2$

Power Factor Analysis

To analyze the problem for calculation of power factor by the finite element method the single-phase run capacitor motor of 100W, 60 Hz, 2 poles of Fig.3 is used.

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Power Factor Calculation by The Finite Element Method. (continued)

The coupled electric circuit used in simulations is shown in Fig.4 where L_{em} and L_{ea} are the main and auxiliary end-winding's inductances and C_{run} the run capacitor. The main and auxiliary windings are input to the finite element software and take into account the winding resistances. The basic theory of a single-phase induction motor shows that, the mean value P_{1ph} of input real power of a connected capacitor and single-phase induction motor can be obtained by the equation (14). The V_T, I_{main}, I_{aux} are the applied voltage, the current of main winding and the current of auxiliary winding, taken in one period T of the time respectively. The time of calculation corresponds to one cycle (1/f) of input power where f is the frequency of applied voltage.

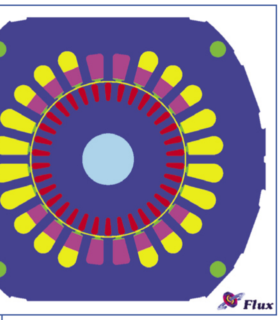


Fig.4. Design of analyzed single-phase motor in Flux 2D.

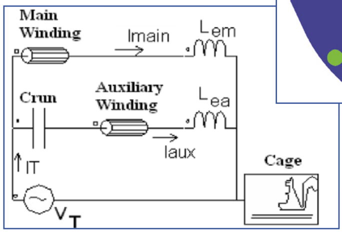
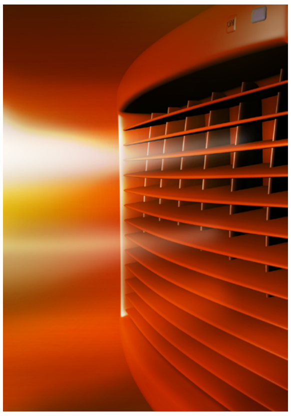


Fig.4. Coupled electric circuit of finite element analysis.

$$P_{1ph} = \frac{1}{T}\int_{t}^{t+T} (V_T x(I_{main} + I_{aux}))dt \qquad (14)$$

It is known that the input power of single-phase induction motors has several harmonics and they are more pronounced in saturated motors where the input current is distorted. The harmonics in the total current leads to a necessary attention for extraction of $\cos\phi_1$. The use of harmonic analysis to calculate the power factor and the displacement factor is not commonly found in publications. However, this methodology is a powerful way to have an accurate result in cases where for example the motor is saturated. In this work the applied voltage is sinusoidal then the equation (13) will be used to evaluate the displacement factor. In Figure 5 we have the experimental and calculated curves of the total motor current, I_T. The harmonic spectrum of these currents is presented in Figure 6. The analysis uses the FFT algorithm that relies on a sample of current taken over the period T at steady state. To calculate the displacement factor by use of spectral analysis, the continuous component of input power and the rms value of the first harmonic of input current are taken from calculations and testing. In order to extract the mean input watts value P_{1ph} from simulations, the V_T, I_{main} and I_{aux} versus time are exported from the post-processor module of software. The mean value of P_{1ph} is obtained directly with $Mean_{value}(P_{1ph}(t)) = P_{1ph}$. Tables II and III present the values taken from simulations and testing to calculate the displacement factor. The time step of the calculation is 0.09 ms and the acquisition time is 0.1 ms. Results obtained by use of spectral analysis are relatively close with the experimental ones.



Conclusion

This work presented the standard methodologies used to calculate the power factor of electrical motors and an extended model considering the harmonics' effects. Finite element approach makes it possible to match the theory. Results of simulations and testing show that the use of harmonics' model provides excellent results for saturated motors.

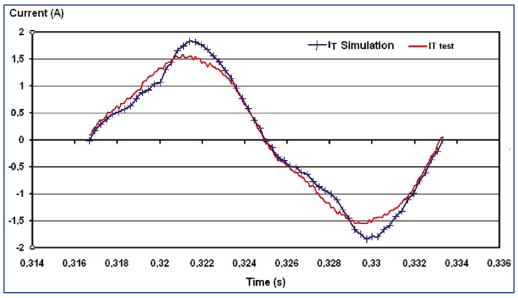


Fig.5. Curves of IT obtained by simulation and test (Steady state).

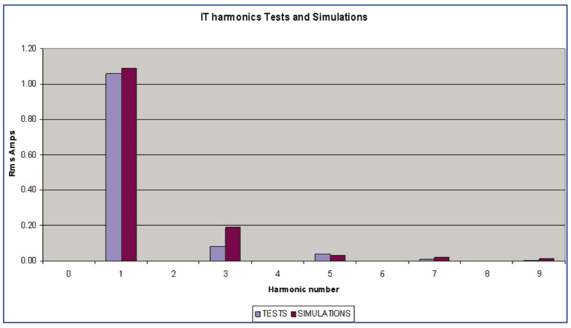


Fig.6. Harmonic spectrum of IT current.

Mean Input Power P_{1ph}	123.2 Watts
RMS Input Current 1st harmonic	1,09 A
RMS Input Voltage 1st harmonic	115 V
Displacement Factor	0.982

Table 2. Displacement factor - Spectral analysis simulation

Mean Input Power P_{1ph}	119.4 Watts
RMS Input Current 1st harmonic	1,06 A
RMS Input Voltage 1st harmonic	115 V
Displacement Factor	0.979

Table 3. Displacement factor - Spectral analysis test