FEM computation of electric dipole fields in stratified conducting media

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Abstract: Modern computing power, and the development of Finite Element Method (FEM) techniques, offer the possibility of deriving numerical solutions for the electromagnetic (EM) fields of an electric dipole located in layered conducting media. The advantage of applying numerical procedures to approximate the mathematical solution is that more general and realistic models of the physical situation can be considered. In this paper, the results obtained using the FLUX 3D FEM technique are compared to the analytical solution for the special case of horizontal interfaces and horizontal dipole. The predictions of both theoretical approaches are compared with the results of electric field experiments.

1 INTRODUCTION

The computation of the propagation effects of extremely low frequency (ELF) electromagnetic (EM) fields in seawater is of prime importance to defence-related research groups interested in the magnetic and electric detection of ships, and in underwater communications between vessels. These EM propagation problems are usually quite complex from a phenomenological point of view, and are governed by Maxwell's equations subject to the boundary conditions imposed by the particular geometry. There are only a few simple ELF problems that have a canonical solution that can be expressed in analytical form. Most practical problems of interest are generally more sophisticated and an analytical solution is often difficult or impossible to find. It is the complexity of the boundary conditions that normally causes the EM problem to be unsolvable analytically.

In many areas of applied EM, numerical techniques have become a major tool. The computing power of today’s computers has increased to the extent that analysis is now possible for problems that were once considered unsolvable. The Finite Element Method (FEM), for example, has become a standard procedure in many EM applications. To our knowledge, however, it has not been used by many authors for predicting the propagation of ELF signals in marine environments, and most works have relied on more conventional techniques. These methods, although working well, are generally limited to environments with parallel interfaces and homogeneous electrical conductivity. In principle, however, the FEM should be applicable to much more complex geometries, such as sloping and/or irregular seabeds, coastal or beach areas, and areas with non parallel layers or variable conductivity. The method should also be applicable to regions that contain scattering bodies with very intricate EM boundary conditions.

As indicated above, few authors have investigated FEM use for ELF propagation problems, and the aim of the present paper was to evaluate the use of one FEM implementation (FLUX 3D) for the fairly simple model geometry of a three-layer conducting half-space. This geometry, which has an uppermost layer of air, represents a standard planar interface ocean and seabed situation. The aim of the report was to assess the usefulness of FLUX 3D for such computations, and then to compare the predictions of the FEM model both with Weaver's[1] analytic solution, and also with experimental results obtained by measuring the propagation from an electric dipole source.

2 MODELLING APPROACH

The present section outlines both the FLUX 3D FEM technique for modelling the effect of the electric dipole, as well as the analytic approach developed by Weaver. The section first describes the geometry used for all of the calculations as well as for the experimental measurements. Although both experiments and calculations were performed at a variety of frequencies, only the results for one frequency (128Hz) will be presented in the present report.

2.1 GEOMETRY AND EXPERIMENT

The experimental results presented later and used for comparison purposes determine the model geometry and parameters used for the various calculations. It is therefore appropriate to start by briefly describing the experiment.

Measurements of the three components of both the magnetic and electric fields produced by a horizontal electric dipole were taken using sensors located on the seabed in 43m of water. The electric fields were measured using in-house designed Ag-AgCl electrode sensors, and a standard commercial (Bartington) fluxgate magnetometer was used to measure the magnetic. A four-metre long horizontal electric dipole located 1m below the sea surface was towed at various orientations and offset distances across the sensors. A power amplifier was used to drive the source at a variety of frequencies. For the 128Hz measurements used in the present report, the source strength was 12.7A-m rms. A photograph of the source suspension system is shown in Figure 1.

Sensor signals were brought to the sea surface and recorded continuously on a small instrumentation platform. Differential global positioning system (DGPS) tracking permitted accurate location of the source's track with respect to the sensors. Conductivity measurements performed after the trials indicated that the sea electrical conductivity was homogeneous and equal to 2.9S/m. The seabed conductivity was assumed to be 0.1S/m.

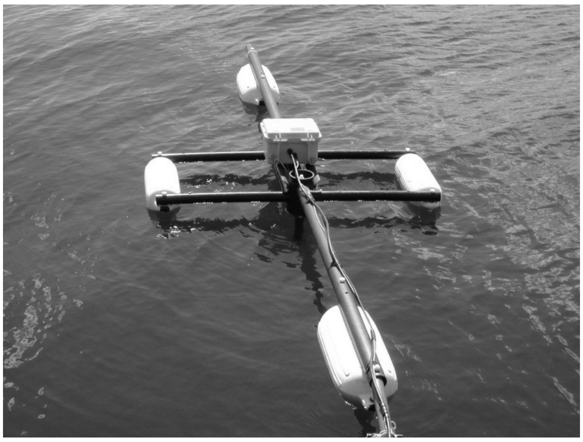

Figure 1. The towed source support assembly complete with DGPS receiver in centre.

2.2 ANALYTIC MODELLING

As indicated earlier, the FEM modelling results were compared with the analytic formalism developed by Weaver. This formalism solves for the fields produced by a point horizontal electric dipole embedded in an infinite parallel-layered air/sea/seabed system such as shown in Figure 2. Field predictions for an extended dipole are calculated by integrating over point dipole constituents. Previous experiments at the Defence Research Establishment Atlantic and elsewhere have shown that the Weaver model agrees with experiment for situations that are well characterised by three layers (i.e. parallel interfaces, and homogeneous conductivity).

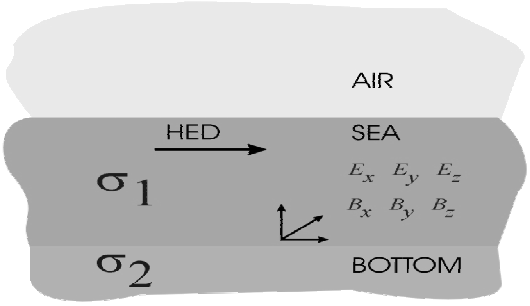

Figure 2. Model geometry.

2.3 FEM MODELLING

FEM modelling of EM problems attempts to solve Maxwell's equations in an approximate manner but with a controllable level of accuracy. The technique ‘discretises’ the domain of the problem into a large number of elementary cells, the corners of which are termed nodes, and where the process of discretisation is termed meshing. The mathematical equations are then solved within each individual cell, and these cell-solutions are assembled to yield an overall solution. The latter is usually performed by solving a linear system of equations.

Such a numerical solution is highly dependent on the characteristic of the mesh used to discretise the computational domain. Up to a point, increasing the mesh resolution results in better accuracy. At some point, however, further increase in mesh resolution has little effect and the solution has converged.

FEM approaches to EM problems are based on the solution of Maxwell's equations expressed in their differential form. These techniques are generally very good at modelling complex inhomogeneous structures, but they have problems modelling open-region geometries. Until quite recently, very few practical modellings of 3-D EM radiation problems were performed using the technique, a fact that resulted for two reasons. Firstly, any practical 3-D problem requires significantly more computation than a 2-D problem, with the result that the method was computationally prohibitive. Secondly, a common difficulty due to spurious solutions, known as 'vector parasites', has only recently been solved [2].

For the present study we utilised the commercial FEM software package FLUX 3D, which is one of the few packages devoted to the 3-D analysis of EM fields in a marine environment.

2.3.1 SOURCE MODELLING IN A CONDUCTING MEDIA

To model the horizontal electric dipole we require a precise knowledge of the near field around an electric source of finite length embedded in a conducting media. To achieve this we make use of two formulations used in the magnetodynamic module of FLUX 3D. The first uses two state variables, the magnetic vector potential **A** and the electric scalar potential V. The second uses the magnetic scalar potential ϕ-ϕ_{red} and the electric vector potential **T,** where ϕ_{red} is termed a *reduced* potential which arises from causes other than any directly imposed sources.[1] Within each formulation, the state variables are related to **B** and **E** by the expressions:

$$(1.a)\quad \mathbf{B}=\nabla\times\mathbf{A} \qquad \mathbf{E}=-j\omega\,\mathbf{A}-\nabla V$$

$$(1.b)\quad \mathbf{H}=\mathbf{T}-\nabla\phi \qquad \sigma\mathbf{E}=\nabla\times\mathbf{T}$$

It is possible to build an electric source using either of these two formulations. By way of example, a point electric dipole can be modelled by a magnetic dipole using the (ϕ, **T**) formulation. The equivalence principle can then be used to turn these solutions for the magnetic dipole into those appropriate to the equivalent electric source. The duality displayed by equations (1.a) and (1.b) in terms of the electric or magnetic fields and sources is self-evident.[2]

In the following calculations, a finite-length antenna was modelled as a thin-wire dipole using the (**A,** V) formulation. The reason for choosing this particular formulation, and using a voltage excitation with an electric scalar potential, was that this formulation lends itself more readily to more complicated antenna representation for future applications. For the frequency and wire length used here, the antenna was assumed to be electrically short. This implies that the antenna has a constant current distribution along its length. (We note that if the distribution of current along the wire were not constant, the current would have to be replaced by an average value. In either case, however, the current is not a direct input parameter to the FEM, and must be calculated from the applied voltage distribution for the particular geometry of the antenna.)

To obtain a unique solution to the problem, it is necessary to constrain the values of the field at all boundary nodes. On the artificial far-boundaries of the computational domain, we therefore imposed Dirichlet infinite boundary conditions.

3 NUMERICAL RESULTS

3.1 INFINITE MEDIUM

As a first step in developing FLUX 3D for use in solving EM propagation problems for complex marine environments, we felt it instructive to obtain solutions for a simple geometry where the analytical solution is well known and understood. As the simplest case is, by far, the infinite conducting medium geometry, we calculated the fields arising from a 2m-long electric wire placed in the centre of a large ‘cube’ of homogeneous seawater. The dipole was located at the origin and oriented along the x-direction. The seawater had electrical conductivity of 2.9S/m, and the wire had an electric source strength of 12.7A-m. A 128Hz sinusoidal voltage excitation was applied.

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[1] E.g. in ferromagnetism ϕ_{red} and ϕ correspond to induced and source-related terms respectively.

[2] (1.a) and (1.b) are mutually transformed by the substitutions $\mathbf{B}\rightarrow j\mathbf{E}/\omega$, $\mathbf{E}\rightarrow\mathbf{H}/\sigma$, $\mathbf{A}\rightarrow j\mathbf{T}/\omega\sigma$ and $V\rightarrow\phi/\sigma$ in (1.a), and $\mathbf{H}\rightarrow\sigma\mathbf{E}$, $\mathbf{E}\rightarrow -j\omega\mathbf{B}$, $\mathbf{T}\rightarrow -j\mathbf{A}\omega\sigma$ and $\phi\rightarrow V\sigma$ in (1.b).

The domain was discretised into 206800 first order tetrahedral finite elements, resulting in a system with approximately 136300 equations. Fields were computed at z=42m and y=20m. Convergence was achieved after 491 iterations. In Figure 3 the results are compared with the fields computed from standard electric dipole equations. As can be seen, the agreement is good. (We note that Bx has not been included in the figure due to its zero value for the infinite media case).

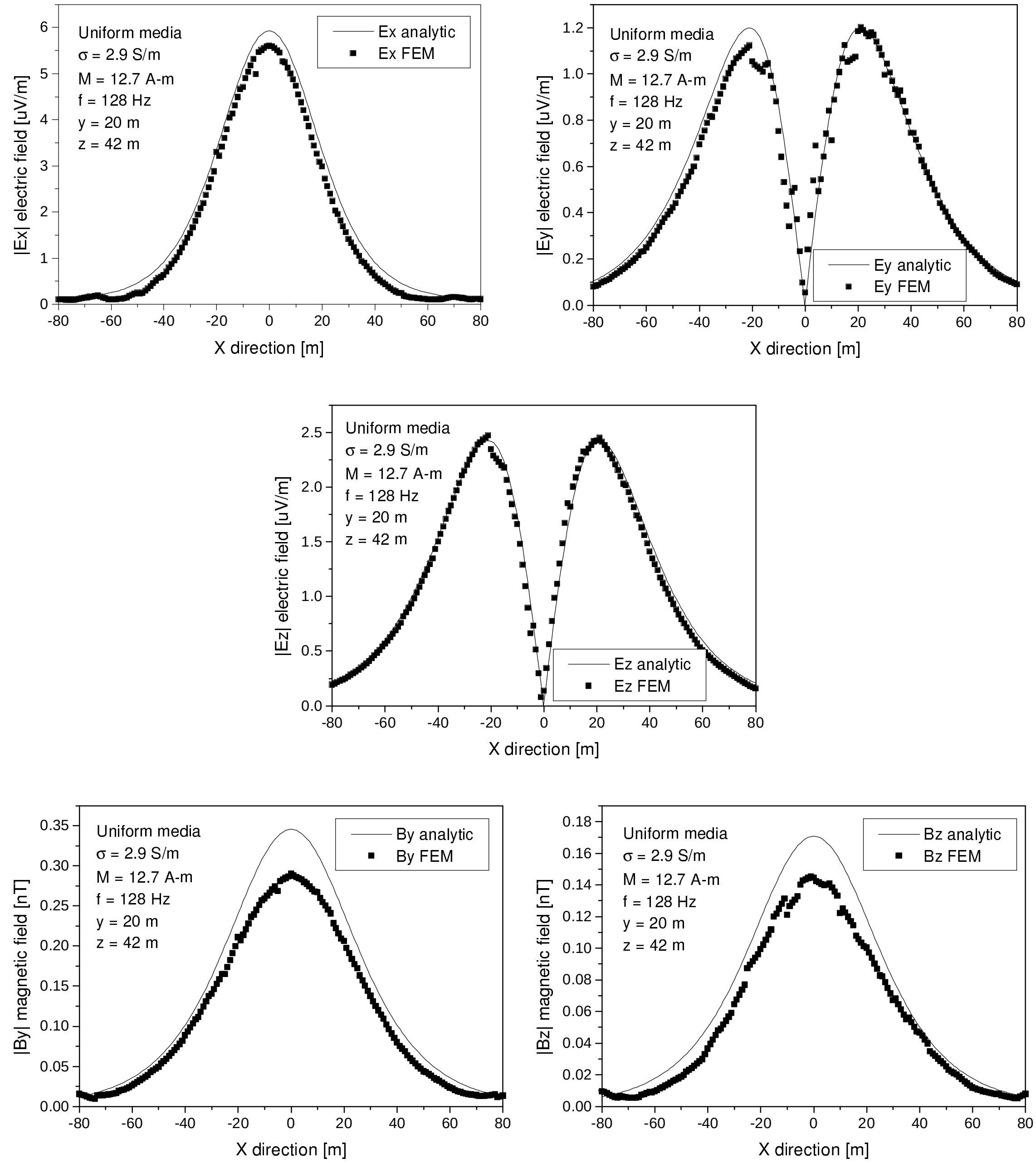

Figure 3. Comparison of FEM and the Weaver model fields for an infinite medium.

3.2 THREE PARALLEL LAYERED MEDIUM

The EM fields were then computed for the three-layer geometry and for the parameters outlined in Section 2.1. The results are compared both with experiment and with the predictions of the Weaver model in Figure 4. The Bx field has again been omitted since these numbers are so small that meaningful comparison is not possible. We note that the FEM fields are computed for x = ±80m whereas the Weaver model and experimental data span a larger distance. This difference arises simply from the size of the domain chosen for the FEM calculations.

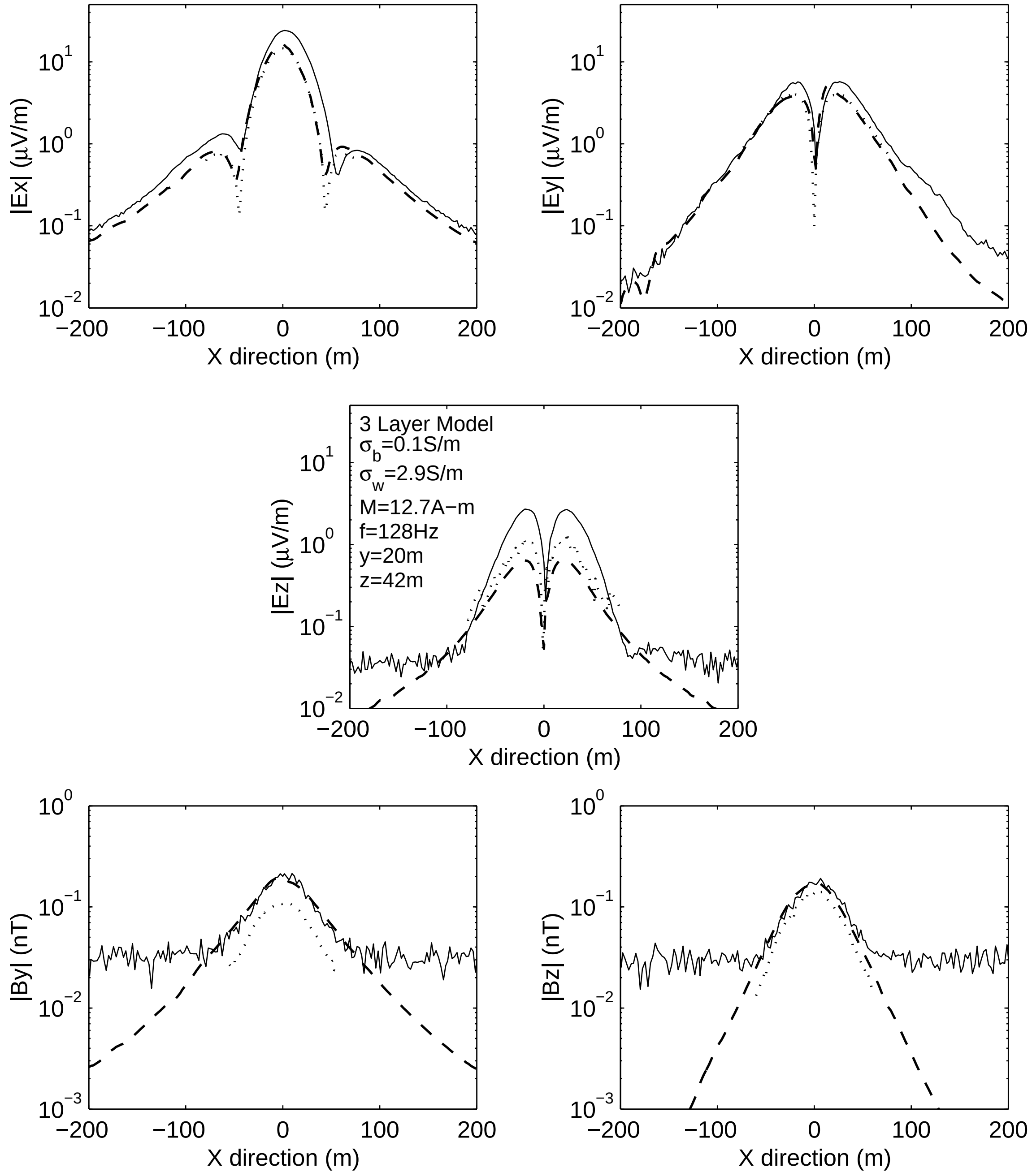

Figure 4. Comparison of FEM (dotted), experimental (solid) and Weaver model (dashed) fields for three-layer medium.

As may be seen, both the agreement between the models and the agreement with experiment is generally good. The only exceptions worth mention are the vertical electric field Ez and the transverse magnetic field By. We believe that the experimental source dipole may have had a slight vertical inclination, which results in Ez being partly in error. The reason for the FEM predicting a low value for By is presently unclear, and will be the study of further investigation.

4 SUMMARY

A method was developed to utilize FEM techniques to calculate the ELF propagation from an electric dipole source. The dipole was implemented in terms of a formulation using a magnetic vector potential **A** and an electric scalar potential V. Calculations were made for the propagation in a three-layered parallel interface air/sea/seabed media. The results for the electric and magnetic field values showed good agreement with both experimental measurement and the predictions of the analytic Weaver model.

One observed disadvantage of the proposed FEM formulation is the process required to implement a finite length electric source. This process could be tedious when modelling an antenna with complicated geometrical structures. A significant advantage of the technique, however, was the comparative simplicity of modifying the media interfaces to include both sloping and irregular surfaces. To this effect, preliminary calculations have already been performed for a sloping seabed and will be reported at later date.

REFERENCES

1. J. T. Weaver, "The Quasi-static Field of an Electric Dipole Embedded in a Two-Layer Conducting Half-Space", 1967, Can. J. Phys. **45**, p1981.
2. K.D. Paulsen and D.R. Lynch, 'Elimination of Vector Parasites in Finite Element Maxwell Solutions', 1991, IEEE Trans. Microwave Theory and Tech., vol. MTT-39, p395.